

source of light, which has been recently investigated by Messrs. Humphreys and Mohler.

The photographs referred to in this communication were taken with a spark source of light placed between the pole pieces of a good electromagnet of the ordinary U-shape, for the use of which I am indebted to the kindness of the Right Rev. Monsignor Molloy. With this magnet I was able to obtain a field of about 25,000 C.G.S. units, and this separated the side lines of the triplets in the violet line 4678 of cadmium to a distance of 0.56 mm. apart. The same separation takes place in the triplet given by the violet line 4680 of zinc, so that in the case of these lines we may infer in round numbers that a magnetic field of 20,000 C.G.S. units strength produces a difference of wave-length of one Ångström unit between the side lines of the triplet, *i.e.*, a separation equal to one-sixth the difference of wave-length between the two D lines of sodium. For iron the separation is about half this amount in the case of several lines.

“Note on the Connection between the Faraday Rotation of Plane of Polarisation and the Zeeman Change of Frequency of Light Vibrations in a Magnetic Field.” By GEO. FRAS. FITZGERALD, F.R.S., F.T.C.D. Received March 2, 1898.

(Being Notes of a Contribution to the Discussion of Mr. Preston's Paper above, read January 20, 1898.)

The rotation of the plane of polarisation of light in a magnetic field is due to the velocity of propagation of light circularly polarised in one direction being different from that of light circularly polarised in the opposite direction. The Zeeman effect is due to a difference in the frequency of vibration of these circularly polarised waves. What is required is to connect the frequency of vibration with the velocity of propagation.\*

All modern theories of dispersion connect these two quantities. The velocity of propagation of light in transparent media is now universally considered to be determined in part by what may be

\* From an abstract of a paper by M. Becquerel, in the ‘Comptes Rendus’ of last year, I understand him to view the Faraday effect as due to a carrying round of the light vibrations by matter rotations. This is quite in accordance with the view sometimes held as to refraction, namely, that it is due to the waves being propagated through the molecules more slowly than through the ether. The dynamical theory of these views is difficult on account of the smallness of the molecules in comparison with the length of the waves. M. Becquerel's view, as I understand it, makes the Faraday effect depend on a change of frequency of rotation of the waves in matter rather than on their velocity of propagation, and is consequently quite at variance both with the commonly received theory and with the one put forward in this note.

called the syntony of the matter and light vibrations, and is consequently dependent on the frequency of the matter vibrations. In most substances the dispersion is controlled within the visible spectrum by a great absorption band in the ultra-violet, this band representing a possible frequency of vibration of the molecules, *i.e.*, of that part of the molecules which affects the ether, be it electrons or something which simulates the actions ascribed to electrons. If owing to any cause this absorption band be changed in position, *i.e.*, the frequency of the molecular vibrations be altered, the dispersion of the medium will be changed, and with it the velocity of propagation of light within the visible spectrum. Now Zeeman has shown that in a magnetic field the frequency of vibration of molecules *producing* light circularly polarised in one direction is different from that of molecules *producing* light circularly polarised in the opposite direction, and that consequently the absorption bands for molecules in a magnetic field for oppositely circularly polarised waves will differ. Hence we conclude that the velocity of propagation of oppositely polarised waves within the visible spectrum will differ, and that is the Faraday effect. Hence these two phenomena are directly connected with one another, independently of any other than general dispersion theory, *i.e.*, independently of any theory of electrons, such as Lorentz has shown will explain the Zeeman effect. If we introduce such a theory, and use it to explain the Faraday effect upon the lines now laid down, we arrive at the interesting conclusion that before the Zeeman effect was observed the Faraday effect would have shown that in the majority of substances we must assume the ether vibrations to be due to the motion of a *negative* electron. In substances with a negative Faraday effect, such as some magnetic bodies, we may conclude either (a) the ether vibrations are due to a *positive* electron, or (b) the absorption band controlling the dispersion is in the ultra-red, or (c) the cause of the Faraday effect may be due rather to a difference of *intensity* of the absorption band for oppositely circularly polarised vibrations than to a difference of *frequency*. This latter alternative is in some ways the most consonant with the usual theory as to the difference between paramagnetic and diamagnetic bodies.

If we proceed to calculate what amount of Faraday effect might be expected from the observed value of the Zeeman effect, we are met by the difficulty of obtaining data. We require to know, for some one or more substances, the amount of the Zeeman effect, the amount of the Faraday effect, and the ordinary dispersion of the substance for light. The only gas (and it is only in gases that the Zeeman effect has been observed) for which the dispersion and Faraday effect are well ascertained, is air, and for it the amount of the Zeeman effect has not been published. In preparation, however,

for the data, it may be interesting to write down shortly the equations which, upon Lorentz's theory as to the cause of the Zeeman effect and Larmor's theory as to dispersion, we may expect to hold.\*

Assuming, then, that the electric displacement in the medium is partly due to the electric force and partly to the displacement of electrons, we may write for its components in the wave face supposed plane and perpendicular to  $z = 0$ ,

$$f = KP + ex, \quad g = KQ + ey,$$

where  $P, Q$  are electric force,  $e$  the electron, and  $x, y$  are coordinates that measure the displacement of this latter. From these and the usual equations connecting magnetic and electric force we get for the medium

$$K\ddot{P} + e\ddot{x} = \frac{d^2P}{dz^2}, \quad K\ddot{Q} + e\ddot{y} = \frac{d^2Q}{dz^2}.$$

For the motion of the matter, if  $m$  be the mass of whatever moves with the electron, matter or effective inertia of ether, and  $H$  be the component of the magnetic force normal to the wave, and  $k$  the coefficient of restitution of the matter displacement which controls its free period,

$$m\ddot{x} + kx = eP + eH\dot{y}, \quad m\ddot{y} + ky = eQ - eH\dot{x}.$$

It is to be observed that if we assume the motion periodic these equations can be reduced to the form that Drude† and Leathem‡ have shown to lead to results that agree with the observations on the effects of magnetised media on the transmission and reflection of light.

If we substitute in these equations what obviously solves them, the equations of a right- or left-handed circularly polarised wave of frequency  $n = p/2\pi$ , and wave-length  $\lambda = 2\pi/q$ , and whose amplitude in  $x$  and  $y$  is  $a$ , and in  $P$  and  $Q$  is  $A$ , we get

$$(KA + ea)p^2 = Aq^2, \\ a(m p^2 \pm eH \cdot p - k) + eA = 0.$$

If we substitute for  $k = mp_0^2$ , where  $p_0$  is a measure of the frequency of the free period corresponding to the forces  $kx, ky$  which have been assumed above to control the motion of the electron, we get for the velocity of propagation  $p/q = V$ ,

$$V^{-2} = K + e \frac{a}{A} \quad \text{and} \quad \frac{a}{A} = \frac{-e}{m(p^2 - p_0^2) \pm eHp}.$$

\* Larmor, 'Phil. Trans.,' A, 1897.

† 'Wied. Ann.,' 1896.

‡ 'Phil. Trans.,' A, 1897.

These enable us to calculate the dispersion of the substance in terms of the difference of velocity of propagation of waves of oppositely circularly polarised light.

To get approximate formulæ in terms of quantities that can be observed, we have the refractive index  $\mu = V_0/V$ , when  $V_0$  is the velocity *in vacuo* and  $p = 2\pi V/\lambda$ . Assuming that  $\lambda_0/\lambda$  is small, *i.e.*, that the dispersion is due to an absorption band far up in the ultra-violet, we get, writing  $\rho$  for  $e/m$ ,

$$\mu^2 = \mu_0^2 + \frac{e\rho\lambda_0^4}{4\pi^2\lambda^2} \pm \frac{e\rho^2\lambda_0^4}{8\pi^3V\lambda} H.$$

The second term on the right-hand side of this equation gives the ordinary dispersion, while the third term gives the Faraday rotation. The first term  $\mu_0$  is given by

$$\mu_0^2 = KV_0^2 + \frac{e\rho\lambda_0^2}{4\pi^2},$$

and is, as Mr. Larmor has pointed out, composed of two parts, the first being essentially refraction and the second dispersion.

In the case of air, it is possible to compare this equation roughly with observation. The equation is of the form

$$\mu^2 = \mu_0^2 + \frac{a}{\lambda^2} \pm \frac{a}{\lambda} \cdot \frac{\rho H}{2\pi V_0}.$$

We may estimate  $a$  from the dispersion in air, and it is approximately  $1.8 \times 10^{-14}$ .

The equation gives for the two absorption bands that exist instead of  $p = p_0$ ,

$$p_0^2 - p_1^2 = \rho p_1 \cdot H, \quad p_0^2 - p_2^2 = \rho p_2 \cdot H,$$

$$\therefore p_2 - p_1 = \rho \cdot H.$$

So that, if  $\delta p$  be the difference of frequency for unit magnetic force,

$$\delta p_0 = \rho.$$

Hence, for the difference of refractive index of two circularly polarised rays, we have

$$\mu \delta \mu = \frac{a}{\lambda} \cdot \frac{\rho H}{2\pi V_0} = \frac{a}{\lambda} \cdot \frac{\delta p_0}{2\pi V_0} \cdot H.$$

Assuming, what is certainly not accurately true, that  $\rho$  is the same for all lines, and taking that Mr. Preston's estimate for some lines of Zn applies to oxygen, namely, that 1 A.U. change of wave-length is produced by a field of 20,000 C.G.S. units of magnetic force, we get—

$$\delta \mu = 4 \times 10^{-14}.$$

The observed rotation in oxygen gives a value for this of  $3.5 \times 10^{-14}$ , so that it is about equal to the calculated value.

The discrepancy may be due to the fact that  $\rho$  is not the same for all spectral lines, and we could not reasonably expect to get an accurate result by assuming that the Zeeman effect in zinc can be used in calculating the Faraday effect in oxygen. On the whole I think the calculation shows that what must be a *vera causa* for a Faraday effect is the whole cause of it. From the mere fact that rotatory polarisation is approximately inversely as the square of the wave-length and consequently vanishes for long waves, it follows that it is essentially a dispersion phenomenon.

“On artificial temporary Colour-blindness, with an Examination of the Colour Sensations of 109 Persons.” By GEORGE J. BURCH, M.A. Communicated by Professor GOTCH, F.R.S. Received February 5,—Read February 17, 1898.

(Abstract.)

By exposing the eye for a sufficient length of time to bright sunlight in the focus of a burning glass behind suitably chosen transparent screens, it is possible to induce over the whole retina a condition of temporary colour-blindness.

After red light the observer is for some minutes completely red-blind, so that scarlet geraniums appear black and roses blue, while yellow flowers seem various shades of green, and purple flowers look violet. The same mistakes are made in sorting Holmgren's wools as by the red-blind.

Temporary violet blindness may be brought about by using a tank of ammonio-sulphate of copper. While it lasts, violet wools look black and purple flowers crimson, but the green foliage appears of a richer tint than usual. The recovery from violet blindness is very slow.

Green blindness may be brought about by exposing the eye to light through three thicknesses of green glass. The colour-scheme of the landscape during this condition is that of a picture painted with vermilion, flake-white, and ultramarine, variously blended.

Purple blindness may be produced by a combination of films stained with magenta and aniline violet, by which the green is absorbed. During purple blindness the vision is practically monochromatic, no colour being visible but green.

If one eye is rendered purple-blind and the other green-blind, the observer sees all objects in their natural colours but with a curiously exaggerated perspective due to the difficulty of combining the images perceived by one eye with those visible to the other.